SINGULAR NUMBERS OF CORRECT RESTRICTIONS
OF NON-SELFADJOINT ELLIPTIC OPERATORS

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Let $\mathcal{L}$ be an elliptic differential expression of the following form: for $u \in C^\infty(\mathbb{R}^n)$

$$(\mathcal{L}u)(x) = \sum_{|\alpha|,|\beta| \leq l} (-1)^{|\alpha|+|\beta|} D^\alpha (A_{\alpha\beta}(x)D^\beta u), \quad x \in \mathbb{R}^n,$$

where $A_{\alpha\beta} \in C^l(\mathbb{R}^n)$ are real-valued functions for all multi-indices $\alpha, \beta$ satisfying $|\alpha|, |\beta| \leq l$. Moreover, let, for a domain $\Omega \subseteq \mathbb{R}^n$, $L_\Omega : D(L_\Omega) \to L_2(\Omega)$ be a linear operator closed in $L_2(\Omega)$ generated by $\mathcal{L}$ on $\Omega$. A restriction $A : D(A) \to L_2(\Omega)$ of $L_\Omega$ is correct if the equation $Au = f$ has a unique solution $u \in D(A)$ for any $f \in L_2(\Omega)$ and the corresponding inverse operator $A^{-1} : L_2(\Omega) \to D(A)$ is bounded.

Let $A$ and $B$ be compact linear operators in a Hilbert space $H$. If there exist $0 < \alpha < \beta$ and $c_1, c_2 > 0$ such that for singular numbers $s_k(A)$ and $s_k(B)$ the conditions $c_1k^{-\alpha} \leq s_k(A), \quad s_k(B) \leq c_2k^{-\beta}$, hold, we say that in the representation $C = A + B$ the operator $A$ is a leading operator and the operator $B$ is a non-leading operator.

**Theorem 1.** Let $l, n \in \mathbb{N}$, $n \geq 2$, $2l(1-\frac{1}{n}) < s \leq 2l$, and $\Omega$ be a bounded domain in $\mathbb{R}^n$ with the boundary $\partial \Omega$ of class $C^{2l}$. Moreover, let $A$ and $B$ be correct restrictions of the operator $L_\Omega$ such that $D(A) \subseteq W_2^s(\Omega), \quad D(B) \subseteq W_2^s(\Omega)$ and the operators $A^{-1} : L_2(\Omega) \to W_2^{-s}(\Omega), \quad B^{-1} : L_2(\Omega) \to W_2^{-s}(\Omega)$ are bounded. Then in the representation $B^{-1} = A^{-1} + K$ the operator $A^{-1}$ is a leading operator and the operator $K = B^{-1} - A^{-1}$ is a non-leading operator.

**Theorem 2.** Let $l, n \in \mathbb{N}$, $n \geq 2$, $2l(1-\frac{1}{n}) < s \leq 2l$ and $\Omega$ be a bounded domain in $\mathbb{R}^n$ with the boundary $\partial \Omega$ of class $C^{2l}$. Then there exists $b > 0$ such that the singular numbers $s_k(B)$ of all correct restrictions $B$ of the operator $L_\Omega$ satisfying the condition $D(B) \subseteq W_2^s(\Omega)$ with the bounded inverse $B^{-1} : L_2(\Omega) \to W_2^s(\Omega)$

$$\lim_{k \to \infty} s_k(B)k^{-\frac{2l}{n}} = b.$$

Also other related results, formulated in [1], will be presented in the talk.