Boundedness and compactness of the integral operators in weighted Sobolev space

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Abstract.

Let $I = (a, b)$, $-\infty \leq a < b \leq \infty$. Let $1 < p, q, r < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$. Suppose that $v, u, \rho$ and $w$ are non-negative and measurable functions on $I$ such that $v^p, u^r, \rho^p, w^q, \rho^{-p'}$ and $w^{-q'}$ are locally summable functions on $I$.

Let $L_{p,\rho} \equiv L_p(\rho, I)$ denote the space of measurable functions on $I$ such that the norm $\|f\|_{p,\rho} \equiv \|\rho f\|_p$ is finite, where $\|\cdot\|_p$ - is the standard norm of the space $L_p(I)$.

We denote by $W_{1,p,r}(u,v) \equiv W_{1,p,r}(u,v,I)$ the set of locally absolutely continuous functions $f$ on $I$ with following finite norm

$$\|f\|_{W_{1,p,r}} = \|uf'\|_r + \|vf\|_p,$$

where $\|\cdot\|_p$ is the standard norm of the space $L_p \equiv L_p(I)$. In case $p = r$ and $u \equiv \rho$ we assume that $W_{1,p}(\rho,v) \equiv W_{1}(\rho,v)$, $\|f\|_{W_{1,p}} = \|f\|_{W_{1}}$.

Let $\tilde{AC}(I)$ be the set of locally absolutely continuous functions with compact supports on $I$. Denote by $\tilde{W}_{1} \equiv \tilde{W}_{1}(\rho,v) \equiv \tilde{W}_{1}(\rho,v,I)$ the closure of the set $\tilde{AC}(I) \cap W_{1}(\rho,v)$ with respect to the norm $\|f\|_{\tilde{W}_{1}} = \|\rho f'\|_p + \|vf\|_p$.

We consider the problem of boundedness and compactness from the weighted Sobolev $\tilde{W}_{1}(\rho,v)$ space into the weighted Sobolev $W_{1,p,r}(u,v)$ space of the integral operators

$$K^+f(x) = \int_a^x K(x,s)f(s)ds, \quad x \in I,$$

$$K^-g(s) = \int_s^b K(x,s)g(x)dx, \quad s \in I$$

with measurable kernel $K(\cdot,\cdot) \geq 0$ on $\Omega = \{(x,s) : a < s \leq x < b\}$.