The study of the Euler equation in vorticity form leads to problems in potential theory, particularly of singular integrals. We intend to cover the basic existence and uniqueness results (in the plane and in the space) in smooth classes and then proceed to study vortex patches, a class of solutions of the vorticity equation in a weak sense. Along the way we will find interesting inequalities for singular integrals. The aggregation equation, which is now attracting a lot of attention, has a similar structure and we will mention how to deal with it using the methods introduced.