INDUCTION ON SCALES IN GEOMETRIC AND HARMONIC ANALYSIS

Induction on Scales is a method for proving functional inequalities which proceeds by induction on the "scale" (or "regularity scale") of the input functions. A very simple example of a suitable functional inequality is the Cauchy–Schwarz inequality on $L^2(\mathbb{R}^n)$, namely

$$\int_{\mathbb{R}^n} f(x)g(x)dx \leq \|f\|_2 \|g\|_2,$$

for nonnegative real functions f, g. A revealing way (albeit far from the simplest way) to prove this inequality is by inducting on "the scale at which f and g are constant". In this short course we will begin by making this example very explicit, and move on to deeper sharp inequalities, such as the Young convolution inequality of Beckner and its far-reaching generalisations of Brascamp and Lieb. This method will be discussed in the context of both discrete and continuous scales, where the latter involves the identification of functionals which vary monotonically as their inputs evolve according to suitable diffusion equations. Finally, if time permits, we will explore the role of induction on scales in combinatorial and oscillatory contexts, drawing powerful conclusions about the euclidean Fourier transform.