ABOUT SCHWARZ LEMMA ON THE BOUNDARY

TUGBA AKYEL

We shall investigate a boundary version of the Schwarz lemma. In a class of holomorphic functions on the unit circle, assuming the existence of angular limit on the boundary point, the estimations below of the modulus of angular derivative will be obtained. We shall consider a function \( f \) holomorphic in the unit disc \( D \), \( f(a) = b \), \( |a| < 1 \) and \( |f(z) - \alpha| < \alpha \) for \( z \in D \), where \( \alpha \) is a positive real number and \( \frac{1}{2} < \alpha \leq 1 \). We shall obtain sharp lower bounds on the modulus of the angular derivative of \( f(z) \) at the point \( c \), where \( f(c) = 2\alpha \), \( |c| = 1 \) by considering the cases \( f(z) - b \) has zeros \( z_1, \ldots, z_n \) in \( D \) which are different from \( z = a \) and has no zeros in \( D \) except \( z = a \).