BRASCAMP-LIEB TYPE INEQUALITIES ON SPHERES

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ABSTRACT. Let $\mathbb{S}^{n-1}$ the unit sphere in $\mathbb{R}^n$ with normalized uniform measure $d\sigma$. Let $f_1, \dots, f_n$ nonnegative measurable functions each depending on a different variable. A direct application of multilinear Hölder's inequality yelds

$$\int_{\mathbb{S}^{n-1}} f_1(x_1)\dots f_n(x_n)d\sigma \leq \|f_1\|_{L^n(\mathbb{S}^{n-1})}\dots\|f_n\|_{L^n(\mathbb{S}^{n-1})},$$

with the constraint of having the same p for all L^p norms on the right-hand side. One can easily make p bigger by continuous embeddings of Lebesgue spaces on the sphere, but it is not clear if p can be smaller than n. In [1] Carlen, Lieb and Loss prove that in the case of n functions each depending on a different variable, one can take $p = 2$ for all dimensions. They also show that this is the sharp exponent for functions with that kind of symmetry.
In this talk I will show how to obtain similar inequalities on spheres for functions that possess other symmetries and I will discuss the sharpness of these results. The proof relies on a monotonicity property of the heat-flow and can be extended to more general compact homogeneous spaces.

REFERENCES

[1] Carlen, E. A., Lieb, E. H., Loss, M., *A sharp analog of Young's inequality on S^N and related entropy inequalities*, J. Geom. Anal., **3**, (2004), 487–520.

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