Conformal moduli and lower homeomorphisms

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We consider the homeomorphisms satisfying at least one of the following conditions

\[ \mathcal{M}_\alpha(f(S_k)) \leq \inf_{\rho \in \text{adm} S_k} \int_G \rho^\alpha(x)Q(x) \, dx, \quad (1) \]

\[ \mathcal{M}_\alpha(f(S_k)) \geq \inf_{\varrho \in \text{extadm} S_k} \int_G \frac{\varrho^\alpha(x)}{Q(x)} \, dx, \quad (2) \]

with a given measurable function \( Q : G \to [0, \infty] \). For such mappings the problem can be formulated somewhat similarly to the classical problem on the properties of solutions to the Beltrami equation \( f_\bar{z} = \mu(z) f_z \), for which the properties of \( f \) are investigated in their dependence on the features of \( \mu \). The main cases in (1)-(2) relate to \( k = 1 \) and \( k = n-1 \), i.e. to moduli of curve and of \((n-1)\)-surface families. We show that inequality (1) yields differentiability a.e., the \((N)\)-property, boundedness of the \( \alpha \)-inner dilatation. We also provide the necessary and sufficient condition for a homeomorphism to satisfy (2). Finally, we establish the relationship between homeomorphisms satisfying (1) for \( k = 1 \) and (2) for \( k = n-1 \).

Joint talk with Ruslan Salimov (Institute of Applied Mathematics and Mechanics, Donetsk, Ukraine)