Abstract. The data clustering problem consists in dividing a data set into prescribed groups of homogeneous data. This is a \(NP\)-hard problem that can be relaxed in the spectral graph theory, where the optimal cuts of a graph are related to the eigenvalues of graph 1-Laplacian. In this paper, we give new notations to punctually describe the paths, among critical eigenvectors of the graph 1-Laplacian, realizing sets with prescribed genus. We introduce the pseudo-orthogonality to characterize the special eigenvalue \(m_3(G)\) for the 1-Laplacian on graphs. Furthermore, we show that \(m_3(G) \geq h_3(G)\), the third graph Cheeger constant. This is a first step for proving that the \(k\)-th Cheeger constant is the minimum of the 1-Laplacian Rayleigh quotient among vectors that are pseudo-orthogonal to the vectors realizing the previous \(k - 1\) Cheeger constants. Eventually, we apply these results to give a method and a numerical algorithm to compute \(m_3(G)\), based on a generalized inverse power method.

Based on joint work with Antonio Corbo Esposito.